Evolution equation:

$$\frac{\partial u}{\partial t}+m^{-\frac{1}{n}}\frac{n}{2n+1}\frac{\partial}{\partial x}\left\{u^{\frac{1}{n}+2}\left\{\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right]^2+\left[\rho gcos\alpha\frac{\partial u}{\partial y}-\gamma\frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right]^2\right\}^{\frac{1}{2n}-\frac{1}{2}}\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right]\right\}$$

$$-m^{-\frac{1}{n}}\frac{n}{2n+1}\frac{\partial}{\partial y}\left\{u^{\frac{1}{n}+2}\left\{\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right]^2+\left[\rho gcos\alpha\frac{\partial u}{\partial y}-\gamma\frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right]^2\right\}^{\frac{1}{2n}-\frac{1}{2}}\left[\rho gcos\alpha\frac{\partial u}{\partial y}-\gamma\frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right]\right\}=0$$

Split into 4 mixed equations:

$$\frac{\partial u}{\partial t}+m^{-\frac{1}{n}}\frac{n}{2n+1}\left[\frac{\partial}{\partial x}(T)-\frac{\partial}{\partial y}(K)\right]=0$$

$$T=u^{\frac{1}{n}+2}\left\{\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial C}{\partial x}\right]^2+\left[\rho gcos\alpha\frac{\partial u}{\partial y}-\gamma\frac{\partial C}{\partial y}\right]^2\right\}^{\frac{1}{2n}-\frac{1}{2}}\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial C}{\partial x}\right]$$

$$K=u^{\frac{1}{n}+2}\left\{\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial C}{\partial x}\right]^2+\left[\rho gcos\alpha\frac{\partial u}{\partial y}-\gamma\frac{\partial C}{\partial y}\right]^2\right\}^{\frac{1}{2n}-\frac{1}{2}}\left[\rho gcos\alpha\frac{\partial u}{\partial y}-\gamma\frac{\partial C}{\partial y}\right]$$

$$C=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\Delta u$$

Integral form over 2D domain **Ω**

$$\int qud\Omega-\int qu^{\prime}\,d\Omega+\Delta tm^{-\frac{1}{n}}\frac{n}{2n+1}\int q\left[\frac{\partial}{\partial x}(T)-\frac{\partial}{\partial y}(K)\right]d\Omega=0$$

$$\int vTd\Omega-\int v[(u)^2]^{\frac{1}{2n}+1}\left\{\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial C}{\partial x}\right]^2+\left[\rho gcos\alpha\frac{\partial u}{\partial y}-\gamma\frac{\partial C}{\partial y}\right]^2\right\}^{\frac{1}{2n}-\frac{1}{2}}\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial C}{\partial x}\right]d\Omega=0$$

$$\int pKd\Omega-\int p[(u)^2]^{\frac{1}{2n}+1}\left\{\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial C}{\partial x}\right]^2+\left[\rho gcos\alpha\frac{\partial u}{\partial y}-\gamma\frac{\partial C}{\partial y}\right]^2\right\}^{\frac{1}{2n}-\frac{1}{2}}\left[\rho g\left(sin\alpha-cos\alpha\frac{\partial u}{\partial x}\right)+\gamma\frac{\partial C}{\partial x}\right]d\Omega$$

$$\int oCd\Omega+\int \nabla o\cdot\nabla ud\Omega=0$$

where *q*, *v*, *p* and *q* are test functions, u' is from the previous time step