3D DEM ANALYSIS OF GRADED ROCK FILL SINKHOLE REPAIR: PARTICLE SIZE EFFECTS ON THE PROBABILITY OF STABILITY

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ABSTRACT

The use of graded rock fill is a cost-effective means of sinkhole repair, providing a stable platform for infrastructure while allowing controlled flow of surface water into the sinkhole throat. Typically, the soil around the sinkhole is excavated down to the rock throat, and large diameter (0.25 to 0.5 m) shot rock aggregate is dumped into the excavation. The shot rock forms a stable bridge over the throat, even though the largest rock particle is often smaller than the throat opening, and additional free draining fill is placed up to grade. This paper investigates the stability of the repair for a range of rock particle diameter (relative to the sinkhole throat diameter). The paper has two goals: a) describe the three-dimensional modeling of the placement of shot rock using the Discrete Element Method, where it is demonstrated that for a given throat radius, there exists an intermediate range of particles sizes for which stability depends upon how the particles arrange during the filling procedure, and b) present a statistical description of this intermediate range of particle sizes for which the throat is semi-stable, using logistic regression to describe the gradual transition from unstable to stable behavior.

Keywords: Discrete Element Method, Sinkhole Repair, Logistic Regression

INTRODUCTION

Problem statement

Shallow sinkholes have been successfully repaired or stabilized by placement of graded rock fill over the exposed sinkhole. Typically, the sinkhole is excavated and large diameter (0.25 to 0.5 m) shot rock aggregate is dumped down the excavated throat of the sinkhole (*1, 2, 3, 4*). Through bridging, the throat is stabilized, and subsequent layers of finer material are placed to create a graded rock fill. This fill provides stability, while still permitting the flow of surface water into the cavity below. In practice, the choice of aggregate size is strictly made based on experience, and Sowers (*4*) recommends that the diameter of the particles in the rock fill be greater than approximately 0.5 the throat width.

In many respects, the rock placement is to be similar to the classic problem of granular material flowing through a hopper or funnel. Particles above a given size will always bridge the throat, even if smaller than the throat diameter, while particles below a given size will never bridge or arch the throat. However, there is an intermediate range of particles for which stability or clogging depends on the order, timing, and how the particles arrange during the filling procedure. This intermediate range, termed the semi-stable state here, is investigated in this paper using 3D discrete element method (DEM) as formulated in the open source code YADE (*5*).

A preliminary study of the stability of a graded rock fill using the DEM was described by Feng et al. (*6*). A series of 2 dimensional DEM simulations for a range of particle diameters demonstrated that a small (relative to the throat diameter) mean particle diameter value will lead to an unstable state, while a large particle diameter will always develop a stable arch, and investigated the intermediate range of particle diameter which can be either stable or unstable depending on the initial random position and diameter of the particles. This paper further develops the approach in 3-dimensions, and has two goals:

a) Describe the three-dimensional modeling of the placement of shot rock using the Discrete Element Method, where it is demonstrated that for a given throat radius, there exists an intermediate range of particles sizes for which stability depends upon how the particles arrange during the filling procedure, and
b) Present a statistical description of this intermediate range of particle sizes for which the throat is semi-stable, using logistic regression to describe the gradual transition from the unstable to stable behavior. This provides a rational method to determine the mean particle size (relative to throat diameter) for a given probability that the repair will be stable.

It is convenient to discuss the stability in terms of a mean diameter ratio, $D_{r,\,mean}$, where

$$D_{r,mean} = \frac{d_m}{d_{throat}}$$

Where d_m = the mean size (diameter) of the shot rock particles
d_{throat} = the diameter of the sinkhole or funnel throat

Thus, the empirical practice of using rock fill with a mean particle size greater than about 0.5 the sinkhole throat width would correspond to a $D_{r,mean} > 0.5$.

Basic assumptions in the Discrete Element Method (DEM)

The Discrete Element Method is a numerical method for computing the motion of a large number of particles such as granular material, where the term "particle" denotes a body that occupies a finite amount of space. Although the DEM particle can be of various shapes, e.g. polyhedral or spherical, in this study, the rock pieces are assumed to be spherical. The DEM was originally proposed by Cundall and Strack (*7*), and makes the following assumptions:
(1) All particles are rigid, but inter-particle deformation is approximated by overlapping between particles using a simple force displacement law.
(2) All overlaps occur in a vanishingly small space in relation to particle sizes.
(3) Single rigid particle motion is predicted by Newton's second law of motion.
A representation of the 2-D particle contact is shown in Figure 1.

Description of the sinkhole model

The geometry of the sinkhole throat and the surrounding overburden soil is shown schematically in Figure 2. Conceptually, the problem is similar to a system of spheres in a funnel. The shape of the funnel is approximated by a series of 8 plates (Figure 3), and different funnel angles characterized by plates inclined angles of 30^o, 45^o or 70^o with respect to the horizontal. For simplicity, the friction between surrounding soil and the rock fill is assumed to be equal, although in the real case they are rarely the same value. A dumping bin composed of four vertical walls and an inclined plate is placed beside the funnel to simulate the unloading of the rock fill from a dump truck (Figure 4), and the plate is inclined 30^o. The random diameter particles are randomly placed inside the bin without overlap.

During the simulation, the particles are specified using the mean diameter d_m and the ratio of maximum diameter to minimum diameter r. Each particle diameter is then randomly generated within the maximum and minimum diameter range of $d_m/(r+1)$, $d_m \cdot r/(r+1)$. Typical physical and geometrical properties for the rock fill are listed in Table 1. All analyses were conducted with a throat diameter $d_{throat}= 1.0$, such that the diameter ratio, $D_{r,mean}$ and the mean particle diameter, d_m have the same value.

SIMULATION PROCEDURE

The simulation included major steps as listed below:
Step 1: Particle size is specified, and particles are randomly generated and placed inside the dumping bin.
Step 2: The dumping bin is opened and the particles are allowed to fall under gravity into the sinkhole.
Step 3: The stable/unstable condition is recorded.

The simulation process was then repeated for a different mean particle diameter and random particle packing. Depending on the particle size, either a stable state is observed such that the particles can successfully form an arch, or an unstable state is observed such that the particles will continuously flow into the funnel throat without forming a stable arch. Examples of both the stable and unstable states for the 45^o funnel are as shown in Figure 5.

DISCUSSION OF RESULTS

Statistical definition of semi-stable state

Feng et al, (6), observed in the 2D simulations that there is an intermediate range of d_m, within which the dumping of the particles can either be stable or unstable due to their initial random position and packing, i.e. there exists an intermediate range of particle diameters, relative to the throat diameter, for which arching may or may not develop, depending on the random nature of the particle sizes and position in the backfill. Simply increasing the particle mean diameter (relative to throat radius) does not provide a stepwise jump from unstable to stable, depending upon how the particles arrange or are dumped. A brief explanation in 2D is shown in Figure 6, where the same three particles are shown to form both a stable state and an unstable state, depending on their initial random positions. Such phenomena dictate that the prediction of the final stable/unstable state must be based on a statistical analysis, i.e. the outcome of the state should be described using a probability function. In structural engineering, a 5-percentile value is often used for the acceptance material properties (*8*), which would correspond for a 95-percentile value for stable sinkhole repair. The mean particle diameter for a 95% probability of stability is investigated here.

Determine the semi-stable mean diameter using binary search method

Feng et al. (*6*) performed a similar stability analysis in 2D, and used a constant value step of mean diameter Δd_m for the next calculation. Such step-by-step or "brute force" method is a time consuming approach to investigate the semi-stable range. In this paper, a binary search method (*9*) is applied during the process listed as follows:

Find a mean diameter value which can guarantee absolute stability, $d_{m1,}$ and a value which is absolute unstable, d_{m2};

(1) Start with the average of $d_{mi}=(d_{m1}+d_{m2})/2$, if d_{mi} value leads to unstable condition, then $d_{m1}=d_{mi}$, if d_{mi} value leads to stable, then $d_{m2}=d_{mi}$;

(2) Repeat Step (2) until the search approximately (see criterion below) where d_{mi} value cannot guarantee either stable or unstable.

The criterion for determining statistically stable/unstable:

> For a calculated mean diameter d_{mi}, run the simulation with different random position/diameter N (e.g. 6) times, if most simulations (e.g. 5 out of 6) are stable, then the d_{mi} value is considered stable, if most simulations (e.g. 5 out of 6) are unstable, then the d_{mi} is considered unstable. When around half of the simulations are either stable or unstable, it is indicating d_{mi} is entering the semi-stable range.

The above binary search method greatly speeds up the search approach. As shown in

Table 2, it only takes 7 trials of d_{mi} to find the semi-stable d_m value around 0.41. The results in Table 2 also demonstrate how the random nature of the particle assembly and filling process produces both stable and unstable configurations for the same value of d_m.

Regression analysis of the probability of the critical particle diameter

A logistic regression (*11*) was performed based on the data obtained using the binary search as described in Section 0 using JMP (*12*), where π is defined as the probability of stability with respect to d_m, and the natural logarithm of the odds (which is referred to as logit) becomes:

$$logit = ln\left(\frac{\pi}{1-\pi}\right)$$

The resulting regression curves in Figure 7 show the transition from unstable to stable as the mean particle diameter increases, for funnel angles of 30^o, 45^o and 70^o. The smooth curve is the predicted probability from the logistic regression, while the circle symbols are the probability of stability from the simulation for a given mean radius, e. g, From Table 2 for d_{mi}=0.44375, there are 4 success runs and 2 failure runs for the mean diameter 0.44375, thus the probability for stability is then 4/6=0.667. This regression suggests that for a 95% probability of a stable sinkhole repair, the mean particle diameter should be 0.468 for the 30^o funnel, 0.461 for the 45^o funnel and 0.475 for the 70^o funnel. The mean diameter ratios for the 3 funnel angle are compared in Figure 8. While additional runs could be performed to better determine the mean diameter as a function of the funnel angle, from a practical perspective the stability can be assumed to be independent of funnel angle. It can be concluded from Figure 8 that to obtain a 95% probability of stable arch, the mean particle diameter or diameter ratio for the 3 funnel angles is about 0.47. This relationship supports the empirical recommendation by Sowers (*4*) of using a diameter ratio greater than 0.5.

CONCLUSIONS

A series of discrete element simulations of the idealized placement of graded rock fill for sinkhole repair were conducted, and the stability was investigated for a range of mean particle diameters relative to the sinkhole throat diameter. The rock particles were idealized as spheres, with a ratio of maximum diameter to minimum diameter of 1.5, with each simulation using a new random assembly of particles. It is shown that there is a large (relative to the throat diameter) mean particle diameter that, although smaller than the throat size, will clog the throat or produce a stable plug. There is a small mean particle diameter which will not lead the formation of a stable plug. The investigation focused on the intermediate range of mean particle diameters between these two values which can be either stable or unstable depending on the initial random position and particle diameters.

Six simulations were performed at each mean particle size and the probability of producing a stable plug was determined. Three different funnel angles were investigated, and it was determined that the mean particle size for a 95% probability of stability was independent of funnel angle, and was about 0.47 times the sinkhole throat diameter, which compares favorably with the empirical value of 0.5.

The discrete element method appears to be a reasonable method to investigate the sinkhole repair procedure. Since the transition in mean particle size from a stable to an unstable assembly of particles is a continuous smooth function rather than a step function, the logistic regression is demonstrated as an appropriate means to estimate the mean size such that a stable repair can be determined with a 95% probability.

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TABLE 1 Geometrical and Physical Properties of the Particles in the Assembly

Sphere Properties		Value	Unit
Mean diameter		0.2-0.5	m
Max diameter/Min diameter Ratio		1.5	
Bottom funnel width/radius (hole Size)		1.0/0.5	m
Density		2650	kg/m^3
Friction Angle	ball-ball	35	
	ball-wall	35	

TABLE 2 Example of running cases for stable/unstable for funnel inclined angle 70°
(Stable=1, Unstable =0, each diameter value d_{mi} with 6 runs)

Trial	Mean Diameter (d_{mi})	Run No.					
		1	2	3	4	5	6
1	0.2	0	0	0	0	0	0
2	0.5	1	1	1	1	1	1
3	0.35	0	0	0	0	0	0
4	0.425	1	1	0	0	1	0
5	0.4625	1	1	1	1	1	1
6	0.44375	1	0	1	0	1	1
7	0.453125	1	1	1	0	0	1

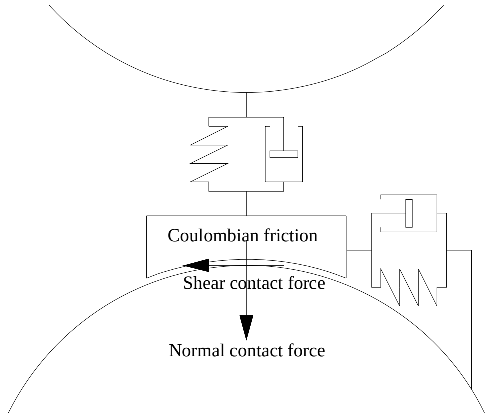


FIGURE 1 Schematic representation of the DEM contact model for normal and shear forces between particles (after Cundall and Strack, 1979)

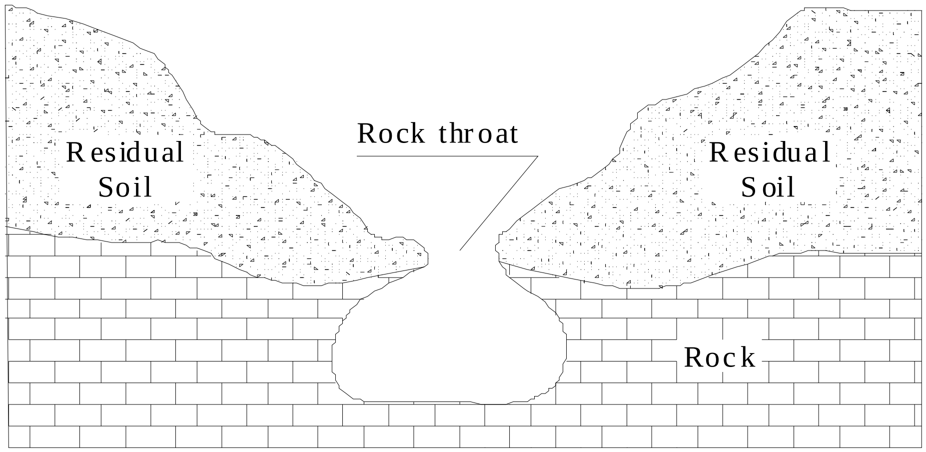
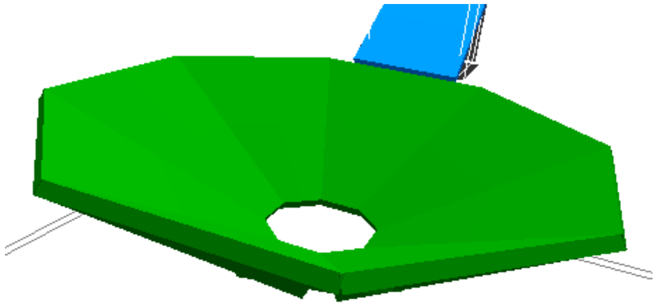
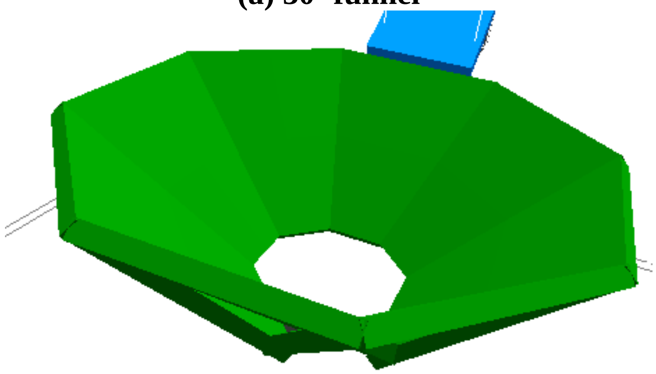


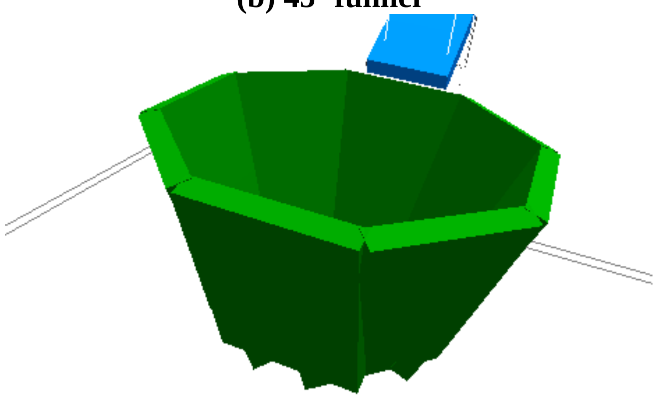
FIGURE 2 Schematic of the sinkhole (Drumm et al. 1990) (*10*)



(a) 30^{o} funnel



(b) 45^{o} funnel



(c) 60^{o} funnel

FIGURE 3 3D funnel with (a) 30^{o}, (b) 45^{o} and (c) 70^{o} to the horizontal plane

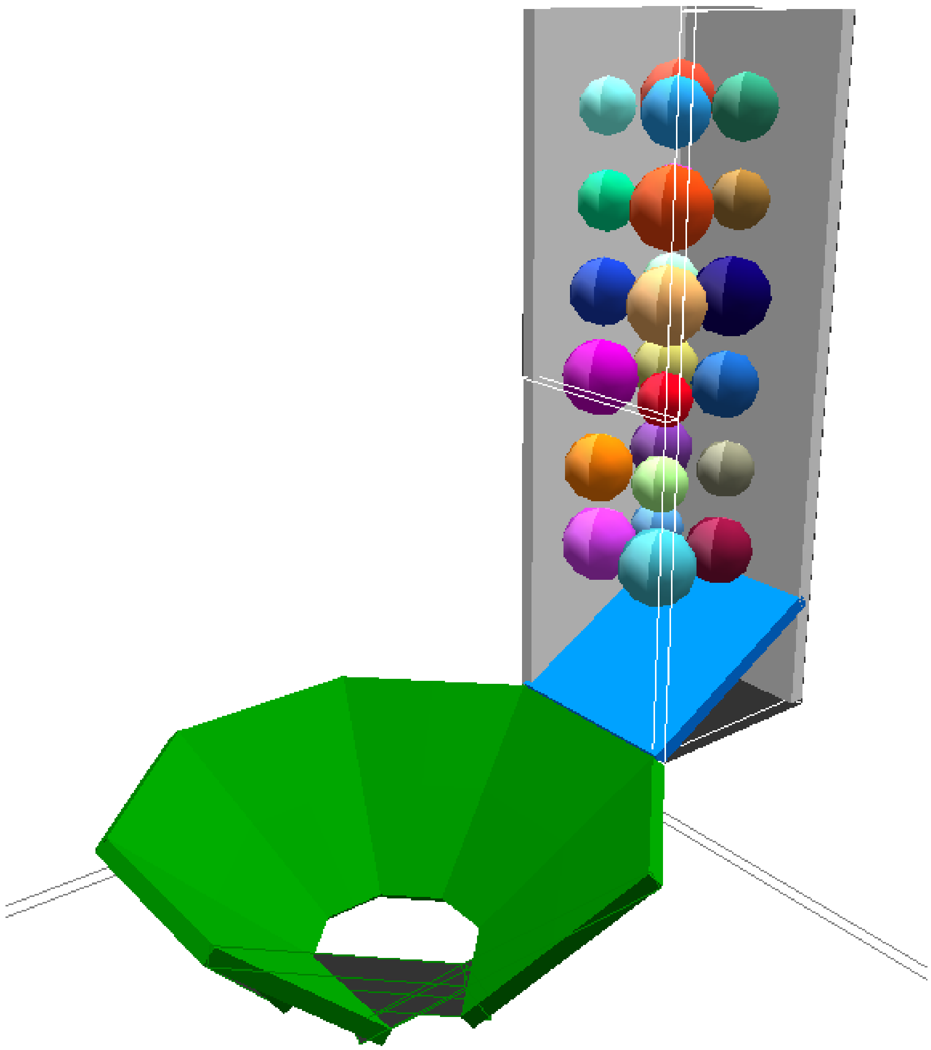
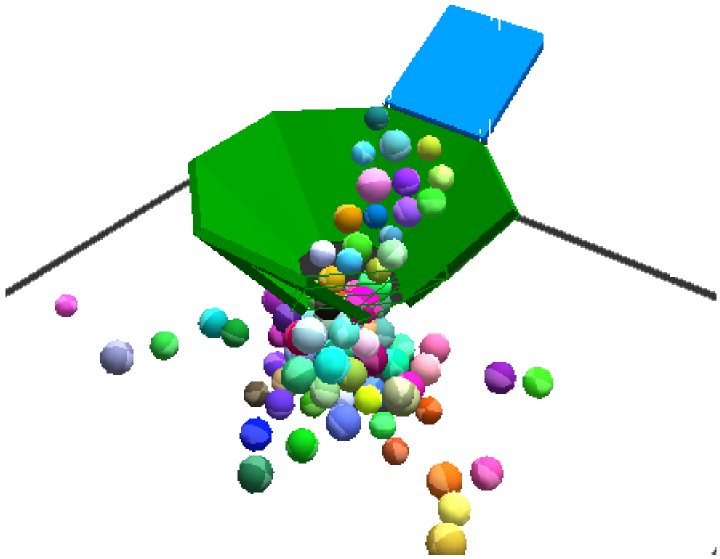
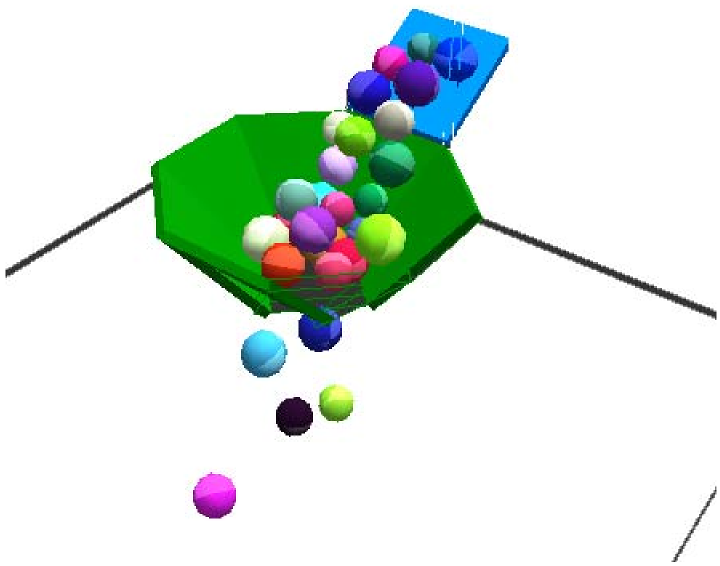


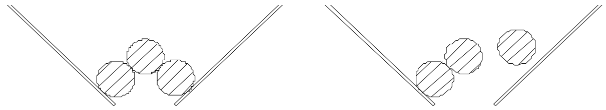
FIGURE 4 Schematic of 3D DEM simulation of the particles through a funnel



(a) Unstable state (small mean particle size relative to throat diameter)

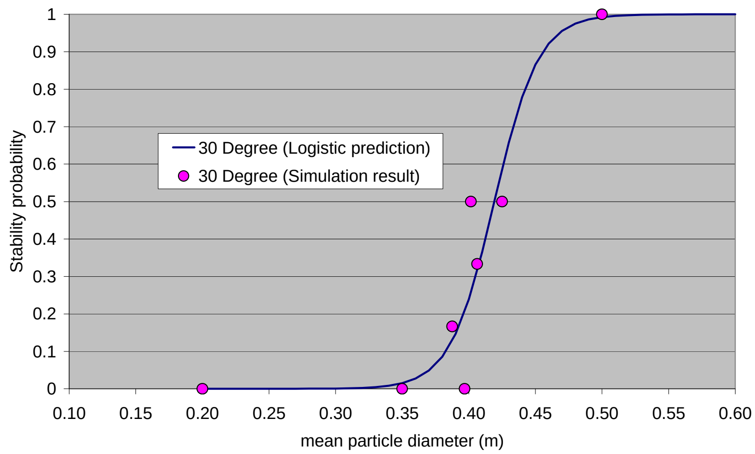


(b) Stable state (large mean particle size relative to throat diameter)
FIGURE 5 Examples of Unstable and Stable state

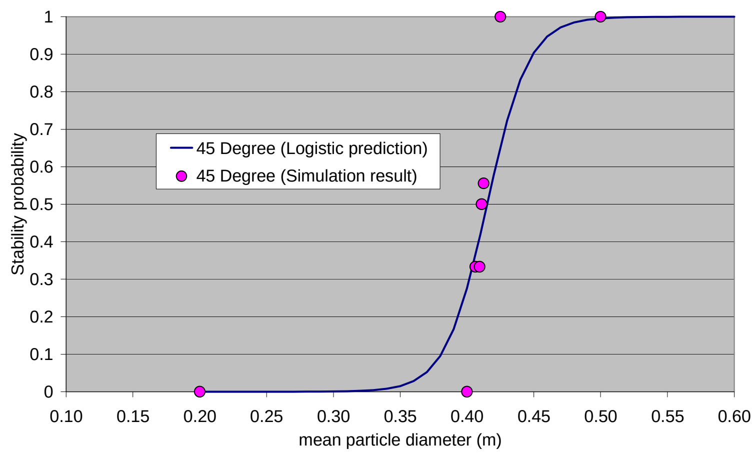


(a) Stable state **(b) Unstable state**

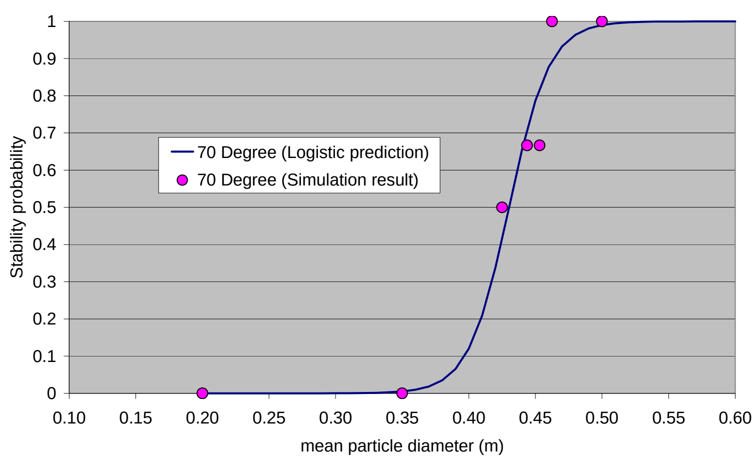
FIGURE 6 Random stable and unstable particle arrangements for the same radius



(a) Funnel angle 30^o



(b) Funnel angle 45^o



c) Funnel angle 70^o

FIGURE 7 Logistic regression plot for simulation results from various funnel inclined angles

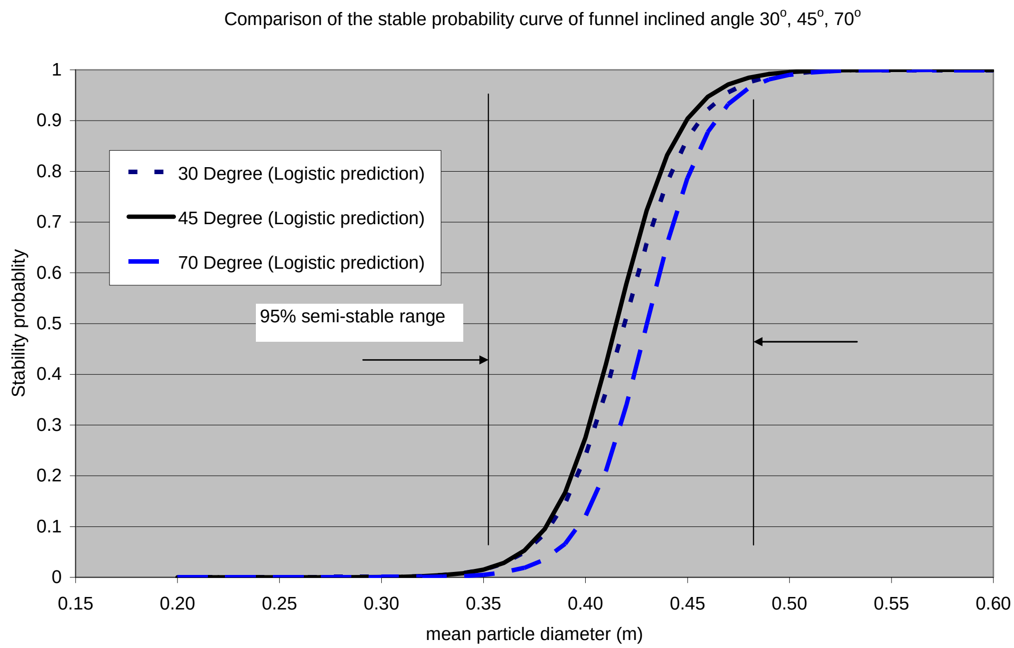


FIGURE 8 Comparisons of the stable probability curve for three funnel angles