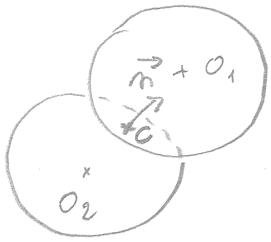
Incremental formulation of contact laws (B. Chareyre)



C = contact point at time "t" = $C(t)$

O_1, O_2 = ref. points at time "t" = $O_1(t)$, $O_2(t)$

The relative displacement between t and $t+\Delta t$ is decomposed in normal$^{(*1)}$ and tangential parts, and corresponds to the relative displacement of material points C_1, C_2, defined as $C_1(t) = C_2(t) = C(t)$

The tangential part reads:

$$\vec{du}_t(t) = \vec{du}(t) - du(t)\,\vec{n}(t)$$

with.

$$\left\{\begin{array}{l} \vec{du}(t) = \underbrace{\left[C_1(t+\Delta t) - C_1(t)\right]}_{dC_1(t+\frac{\Delta t}{2})} - \underbrace{\left[C_2(t+\Delta t) - C_2(t)\right]}_{dC_2(t+\frac{\Delta t}{2})} \\[2ex] dC_i(t) = \vec{dO_i}(t+\tfrac{\Delta t}{2}) + \overset{\text{matrix}}{\overset{\downarrow}{dR_i}}(t+\tfrac{\Delta t}{2})\cdot\vec{O_iC}(t+\tfrac{\Delta t}{2}) \quad (*^2) \\[1ex] \qquad = \vec{V_i}(t+\tfrac{\Delta t}{2})\,\Delta t + \left(\dot{R}_i(t+\tfrac{\Delta t}{2})\,\Delta t - Id\right)\vec{O_iC}(t+\tfrac{\Delta t}{2}) \end{array}\right.$$

where $\vec{dO_i}$, dR_i are the translation/rotation of body "i" between $t+\Delta t$ and t.

The increment of contact force produced by $\vec{du}_t$ is $\vec{dF}_t = -k_s \vec{du}_t$, giving the new force

$$\left\{\begin{array}{ll} \vec{F}_t \mathrel{+}= \vec{dF}_t & (1) \quad (*^3) \\[2ex] \|\vec{F}_t\| \leq \text{shear\_strength} & (2) \end{array}\right.$$

(1) and (2) applied sequencialy.

(*1) This incremental definition of the shear forces and shear displacements could be used for the normal part:

$$\begin{cases} \overrightarrow{du_n} = (\overrightarrow{du} \cdot \vec{n})\,\vec{n} \\ \vec{u}_n \mathrel{+}= \overrightarrow{du_n} \\ \vec{f}_n \mathrel{+}= k_n \overrightarrow{du_n} \end{cases}$$

but u_n can be defined <u>exactly</u> using current positions. This direct approach is prefered in the algorithm.

(*2) If M is a rotation matrix, the displacement $\overrightarrow{dP}$ of point $\vec{P}$ in this rotation is $M.\vec{P} - \vec{P} = (M - Id)\,\vec{P}$

Here, $M = R(t+\Delta t) - R(t) = \dot{R}(t+\frac{\Delta t}{2}) \cdot \Delta t$.

If M is a small rotation, the product $(M-Id)\vec{P}$ can be approximated by a cross-product

$$\overrightarrow{dP} \simeq \vec{\Omega} \times \vec{P}, \quad \text{with } \|\vec{\Omega}\| = \text{rotation angle}$$

$$\text{and } \vec{n}_\Omega = \frac{\vec{\Omega}}{\|\vec{\Omega}\|} \quad \text{the axis of rotation.}$$

This approximation is used for faster computations only.

With quaternions, the equations would read:

$$\overrightarrow{dP} = q \cdot \vec{P} \cdot q^{-1} - \vec{P}$$

(or in C++ : $\overrightarrow{dP} = q.\vec{P} - \vec{P}$)

(*3) Note that the contact force $\vec{F}$, hence its part $\vec{F_t}$, is also updated to account for the "global" (i.e. rigid body-like) rotation of interacting bodies.

Again, the equivalence between rotation and cross-product is used to do this update:

Let $\vec{\Omega_1}$ et $\vec{\Omega_2}$ be the rotations vectors of b_1, b_2.

We define the "global" rotation as

$$\vec{\Omega}_{12} = \vec{\Omega}_{12}^{n} + \vec{\Omega}_{12}^{t}$$

$\vec{\Omega}_{12}^{n}$: rotation around contact normal

$\vec{\Omega}_{12}^{t}$: rotation around an arbitrary axis of the contact plane.

$$\vec{\Omega}_{12}^{n} = \left(\frac{(\vec{\Omega}_1 + \vec{\Omega}_2)}{2} \cdot \vec{n}\right) \cdot \vec{n}$$

$$\vec{\Omega}_{12}^{t} = \underbrace{\|\vec{n}(t) \times \vec{n}(t-\Delta t)\|}_{\text{rotation angle.}} \times \underbrace{\frac{\vec{n}(t) \times \vec{n}(t-\Delta t)}{\|\vec{n}(t) \times \vec{n}(t-\Delta t)\|}}_{\text{unit vector axis of rotation}} = \vec{n}(t) \times \vec{n}(t-\Delta t)$$